Finite Automata

Motivation

An Example

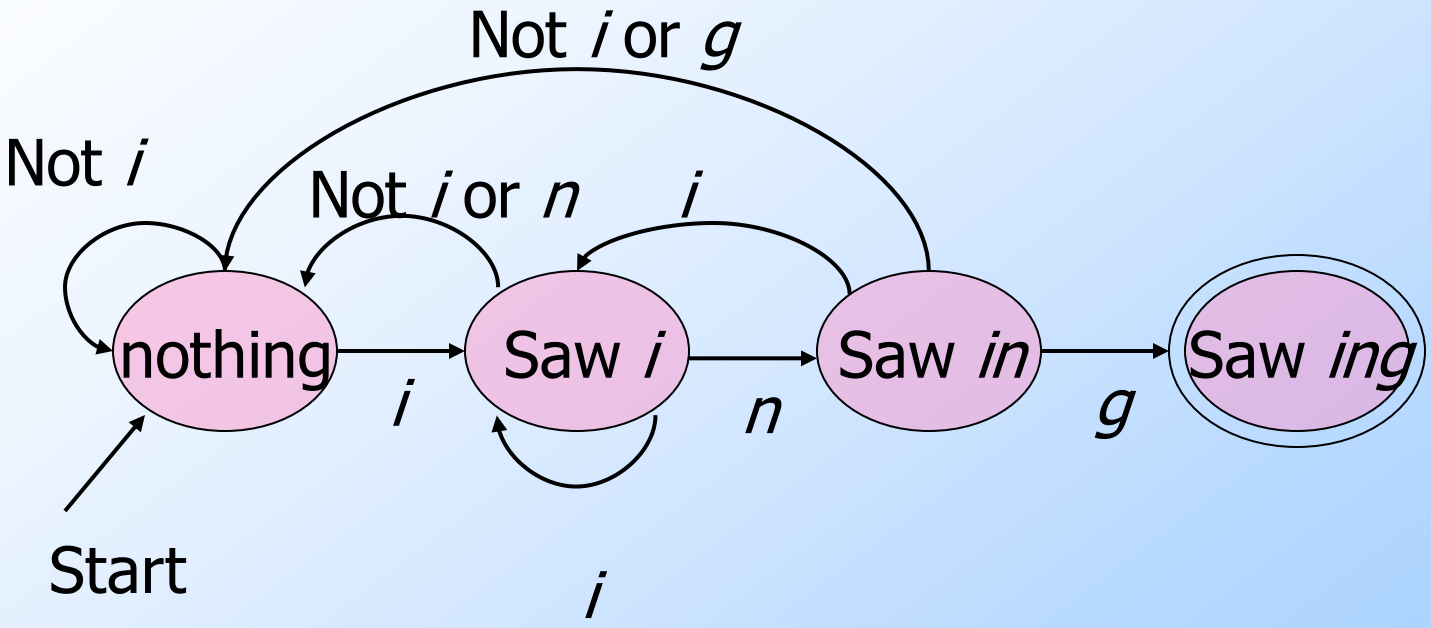
Informal Explanation

- Finite automata are finite collections of states with transition rules that take you from one state to another.
- Original application was sequential switching circuits, where the "state" was the settings of internal bits.
- Today, several kinds of software can be modeled by FA.

Representing FA

- Simplest representation is often a graph.
  - Nodes = states.
  - Arcs indicate state transitions.
  - Labels on arcs tell what causes the transition.

Example: Recognizing Strings Ending in "ing"



Automata to Code

- In C/C++, make a piece of code for each state. This code:
  1. Reads the next input.
  2. Decides on the next state.
  3. Jumps to the beginning of the code for that state.

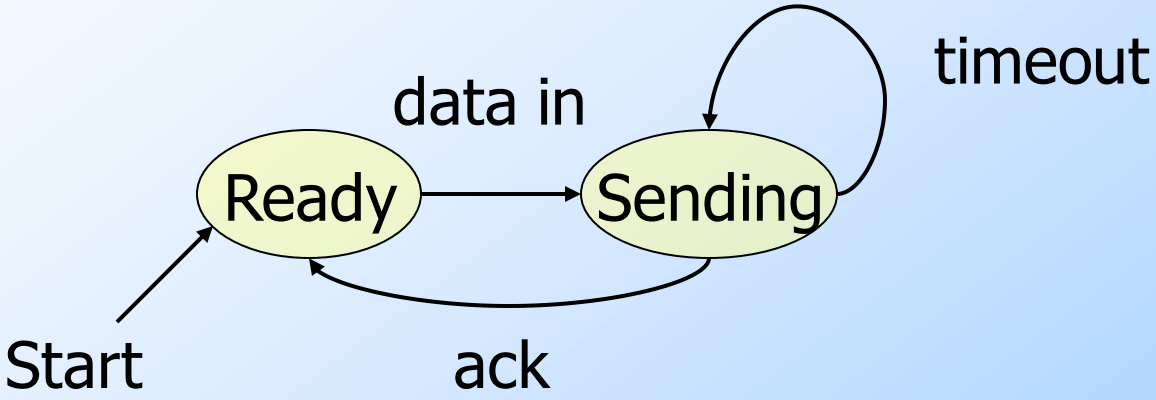
Example: Automata to Code

```
2: /* i seen */
  c = getNextInput();
  if (c == 'n') goto 3;
  else if (c == 'i') goto 2;
  else goto 1;
3: /* "in" seen */
  . . .
```

Automata to Code – Thoughts

- How would you do this in Java, which has no goto?
- You don't really write code like this.
- Rather, a code generator takes a "regular expression" describing the pattern(s) you are looking for.
  - Example: `.*ing` works in grep.

Example: Protocol for Sending Data



Extended Example

- Thanks to Jay Misra for this example.
- On a distant planet, there are three species, a, b, and c.
- Any two different species can mate. If they do:
  1. The participants die.
  2. Two children of the third species are born.

Strange Planet – (2)

- Observation: the number of individuals never changes.
- The planet *fails* if at some point all individuals are of the same species.
  - Then, no more breeding can take place.
- *State* = sequence of three integers – the numbers of individuals of species a, b, and c.

Strange Planet – Questions

- In a given state, must the planet eventually fail?
- In a given state, is it possible for the planet to fail, if the wrong breeding choices are made?

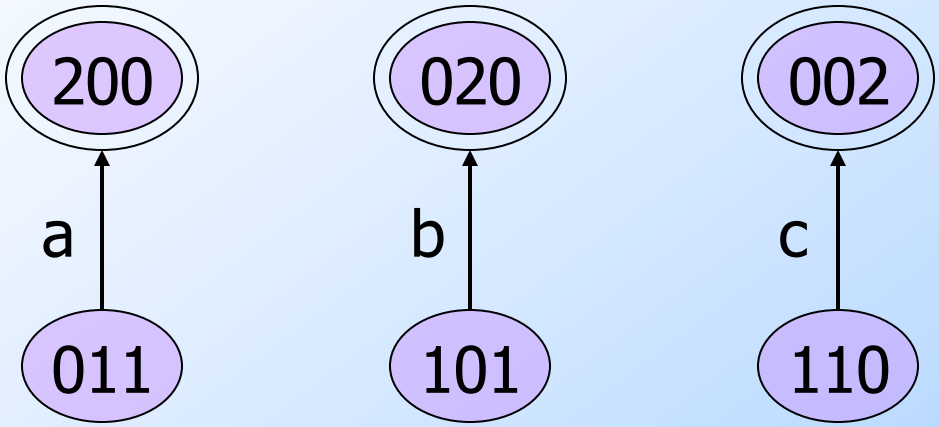
Questions – (2)

- These questions mirror real ones about protocols.
  - "Can the planet fail?" is like asking whether a protocol can enter some undesired or error state.
  - "Must the planet fail" is like asking whether a protocol is guaranteed to terminate.
    - Here, "failure" is really the good condition of termination.

Strange Planet – Transitions

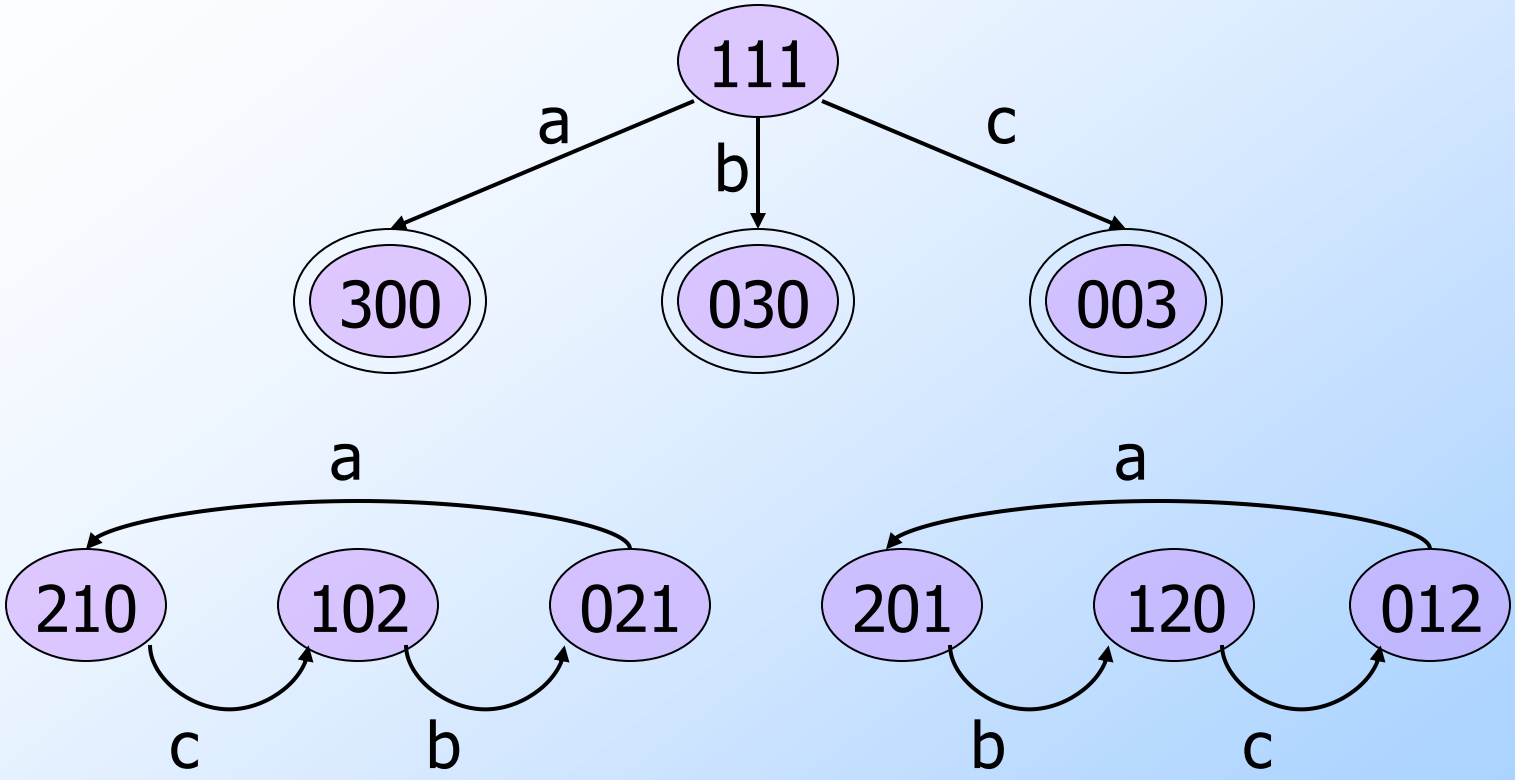
- An *a-event* occurs when individuals of species b and c breed and are replaced by two a's.
- Analogously: b-events and c-events.
- Represent these by symbols a, b, and c, respectively.

Strange Planet with 2 Individuals



Notice: all states are "must-fail" states.

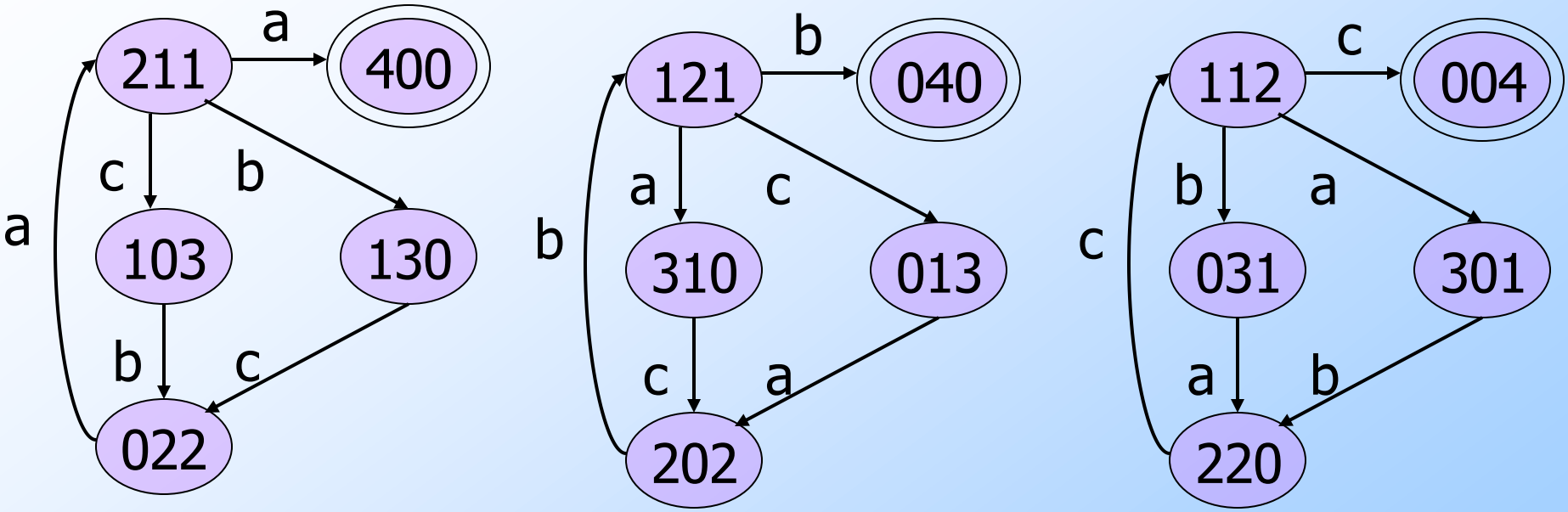
Strange Planet with 3 Individuals



Notice: four states are "must-fail" states.
The others are "can't-fail" states.

State 111 has several transitions.

Strange Planet with 4 Individuals

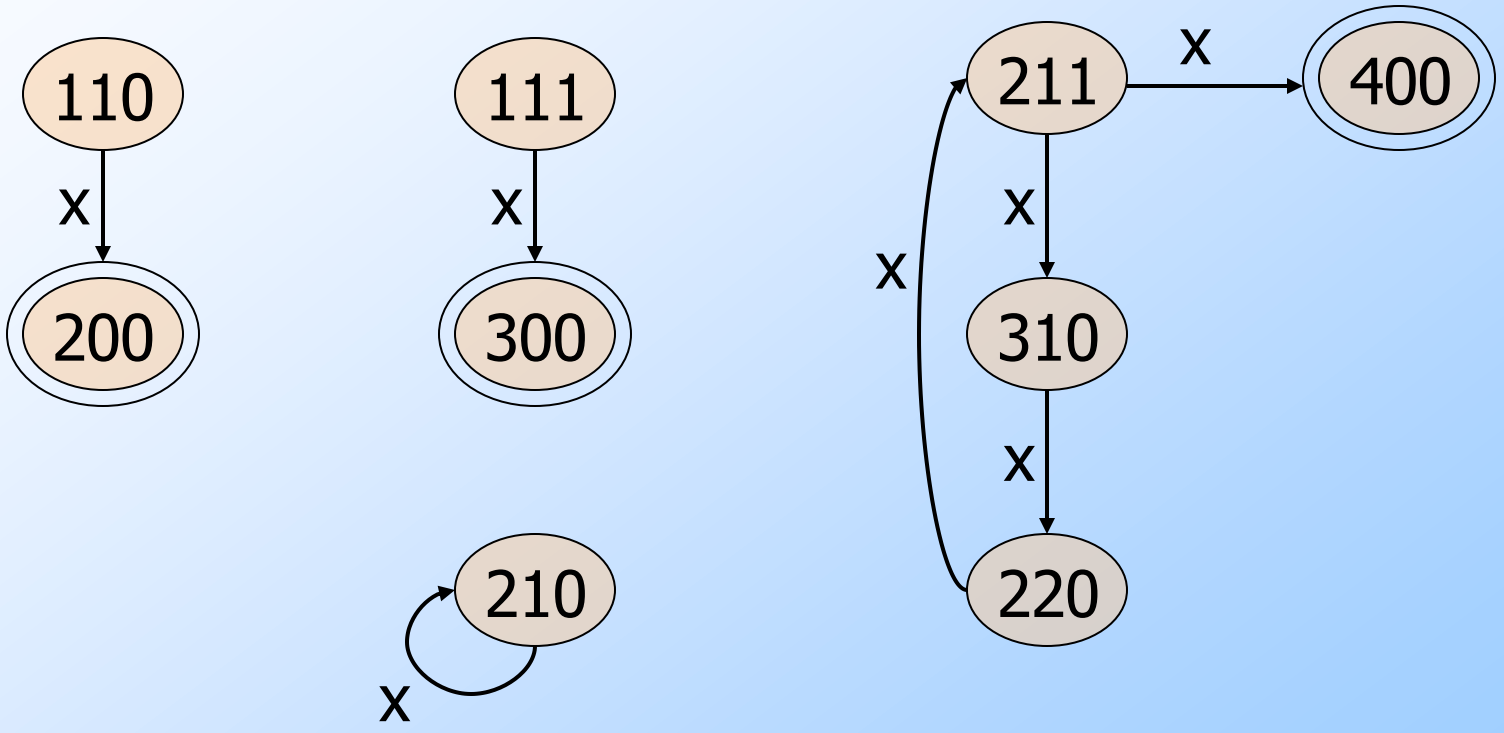


Notice: states 400, etc. are must-fail states.
All other states are "might-fail" states.

Taking Advantage of Symmetry

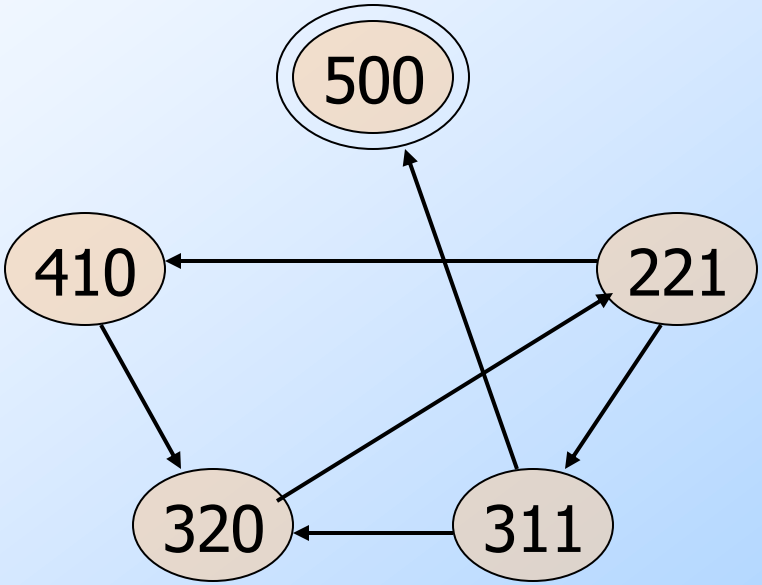
- The ability to fail depends only on the *set* of numbers of the three species, not on which species has which number.
- Let's represent states by the list of counts, sorted by largest-first.
- Only one transition symbol, *x*.

The Cases 2, 3, 4



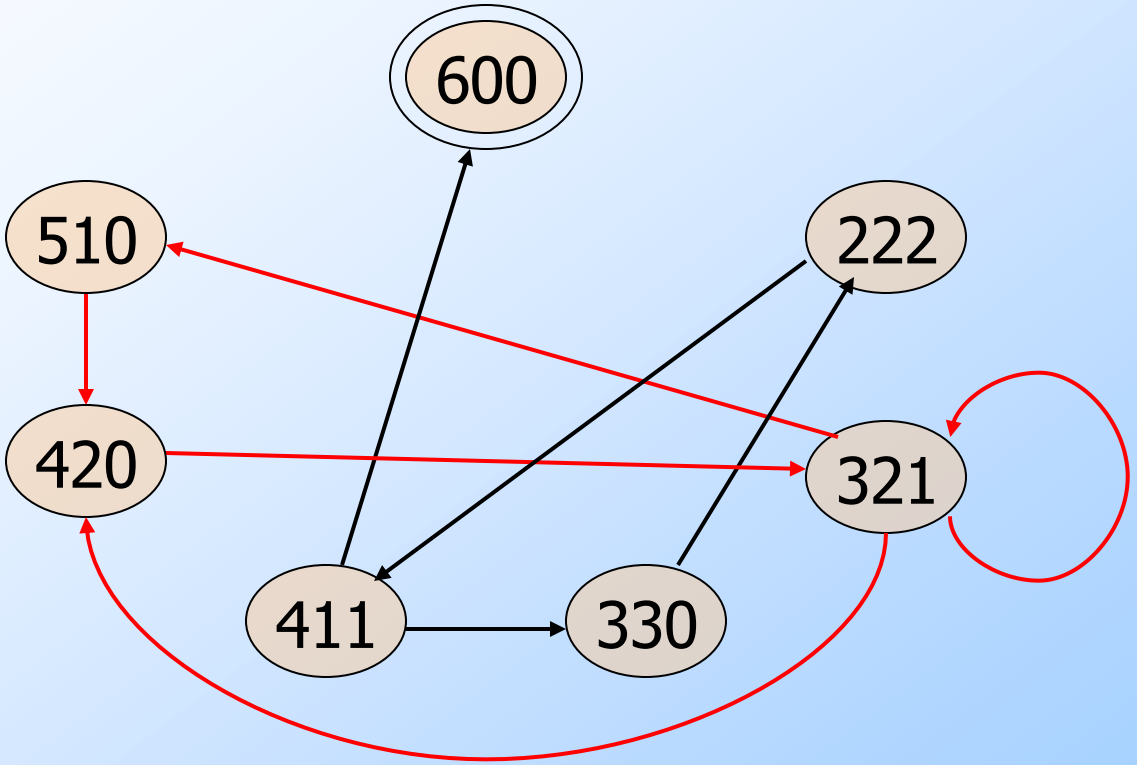
Notice: for the case n = 4, there is *nondeterminism* : different transitions are possible from 211 on the same input.

5 Individuals



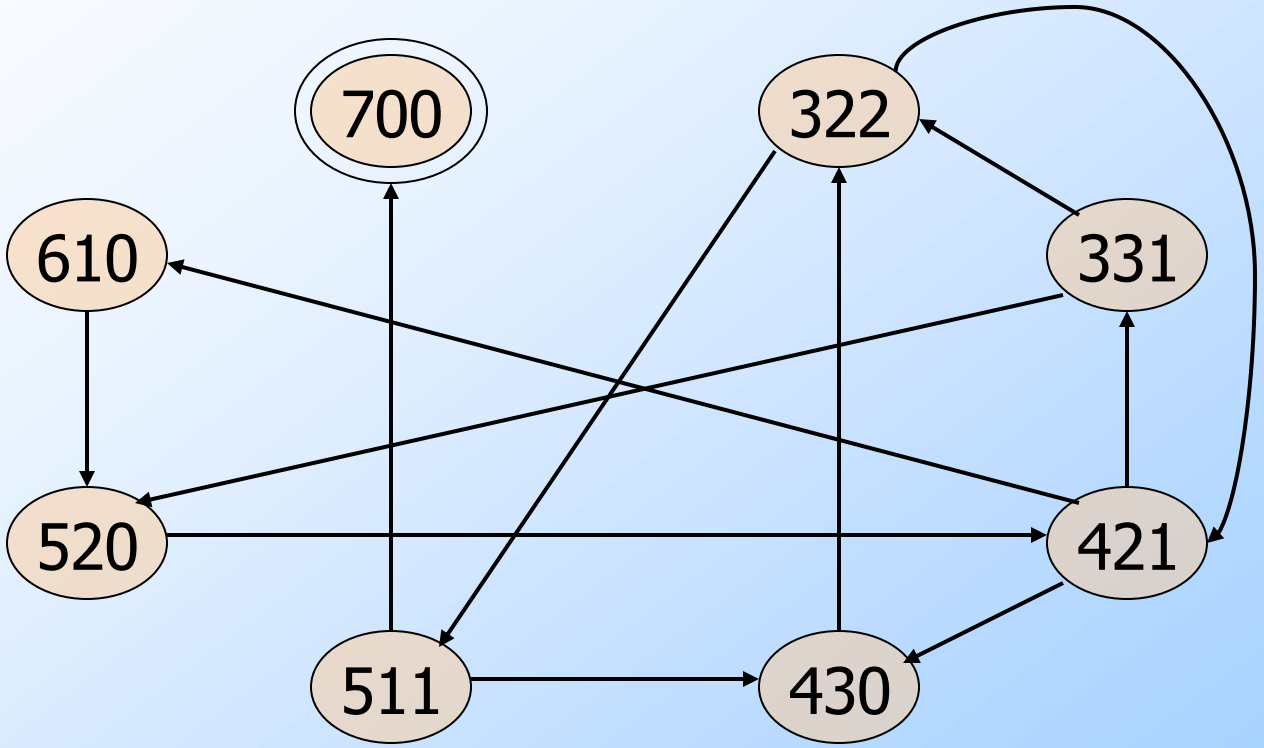
Notice: 500 is a must-fail state; all others are might-fail states.

6 Individuals



Notice: 600 is a must-fail state; 510, 420, and 321 are can't-fail states; 411, 330, and 222 are "might-fail" states.

7 Individuals



Notice: 700 is a must-fail state; All others are might-fail states.

Questions for Thought

1. Without symmetry, how many states are there with *n* individuals?
2. What if we use symmetry?
3. For *n* individuals, how do you tell whether a state is "must-fail," "might-fail," or "can't-fail"?