Classification of Signals & Systems

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WHAT IS SYSTEM?

- Systems process input signals to produce output signals
- A system is combination of elements that manipulates one or more signals to accomplish a function and produces some output.

Types of Systems

- Causal & Anticausal
- Linear & Non Linear
- Time Variant &Time-invariant
- Stable & Unstable
- Static & Dynamic
- Invertible & Inverse Systems

Causal & Anticausal Systems

- Causal system : A system is said to be *causal* if the present value of the output signal depends only on the present and/or past values of the input signal.
- Example: $y[n]=x[n]+1/2x[n-1]$

Causal & Anticausal Systems Contd.

- Anticausal system : A system is said to be *anticausal* if the present value of the output signal depends only on the future values of the input signal.

- Examples of causal systems:

$$y(t) = \frac{1}{C}\int_{-\infty}^{t} x(\tau)d\tau \qquad \text{or} \qquad y[n] = x[n-1].$$

- Examples of non-causal systems:

$$y(t) = x(-t) \qquad \text{or} \qquad y[n] = \frac{1}{3}\big(x[n-1] + x[n] + x[n+1]\big).$$

Linear & Non Linear Systems

1.7.1 Linear and Nonlinear Systems

A system is said to be linear if Superposition theorem applies to that system. Consider the two systems defined as follows :

$y_1(t) = f[x_1(t)]$ i.e. $x_1(t)$ is excitation and $y_1(t)$ is response.

$y_2(t) = f[x_2(t)]$ i.e. $x_2(t)$ is excitation and $y_2(t)$ is response.

Then for the linear system,

$$f[a_1 x_1(t) + a_2 x_2(t)] = a_1 y_1(t) + a_2 y_2(t) \quad \text{... (1.7.1)}$$

Here a_1 and a_2 are constants.

(ii) $y(t) = x^2(t)$

The output of the system to two inputs $x_1(t)$ and $x_2(t)$ becomes,

$$y_1(t) = f[x_1(t)] = x_1^2(t)$$

$$y_2(t) = f[x_2(t)] = x_2^2(t)$$

Hence linear combination of these outputs become,

$$\begin{aligned} y_3(t) &= a_1 y_1(t) + a_2 y_2(t) \\ &= a_1 x_1^2(t) + a_2 x_2^2(t) \end{aligned}$$

Now let us find the response of the system to linear combination of inputs. i.e.,

$$\begin{aligned} y_3'(t) &= f[a_1 x_1(t) + a_2 x_2(t)] \\ &= [a_1 x_1(t) + a_2 x_2(t)]^2 \\ &= a_1^2 x_1^2(t) + a_2^2 x_2^2(t) + 2 a_1 a_2 x_1(t) x_2(t) \end{aligned}$$

Here note that $y_3'(t) \neq y_3(t)$. Hence this is not linear system.

Time Invariant and Time Variant Systems

- A system is said to be *time invariant* if a time delay or time advance of the input signal leads to a identical time shift in the output signal.

$$y_i(t) = H\{x(t - t_0)\}$$

$$= H\{S^{t0}\{x(t)\}\} = HS^{t0}\{x(t)\}$$

$$y_0(t) = S^{t0}\{y(t)\}$$

$$= S^{t0}\{H\{x(t)\}\} = S^{t0}H\{x(t)\}$$

Solution : (i) $y(t) = \sin x(t)$

Let us determine the output of the system for delayed input $x(t-t_1)$. i.e.,

$$y(t, t_1) = f[x(t-t_1)]$$

$$= \sin x(t-t_1) \quad \text{... (1.7.6)}$$

Here $y(t, t_1)$ represents output due to delayed input.

Now delay the output $y(t)$ by t_1. Hence we have to replace t by $t-t_1$ in $y(t) = \sin x(t)$. i.e.,

$$y(t-t_1) = \sin x(t-t_1)$$

On comparing the above equation with equation 1.7.6 we find that,

$$y(t, t_1) = y(t-t_1)$$

This satisfies equation 1.7.5. Hence the system is time invariant.

Stable & Unstable Systems

- A system is said to be *bounded-input bounded-output stable* (BIBO stable) iff every bounded input results in a bounded output.

i.e.

$$\forall t \quad |x(t)| \le M_x < \infty \rightarrow \forall t \quad |y(t)| \le M_y < \infty$$

Stable & Unstable Systems Contd.

Example

- $y[n]=1/3(x[n]+x[n\text{-}1]+x[n\text{-}2])$

$$y[n]=\frac{1}{3}\left|x[n]+x[n-1]+x[n-2]\right|$$

$$\leq\frac{1}{3}(|x[n]|+|x[n-1]|+|x[n-2]|)$$

$$\leq\frac{1}{3}(M_x+M_x+M_x)=M_x$$

Stable & Unstable Systems Contd.

Example: The system represented by

$$y(t) = A\,x(t) \text{ is unstable ; } A>1$$

Reason: let us assume x(t) = u(t), then at every instant u(t) will keep on multiplying with A and hence it will not be bonded.

Static & Dynamic Systems

- A static system is memoryless system
- It has no storage devices
- its output signal depends on present values of the input signal
- For example

$$i(t) = {}^{1}\!/_{R}\, v(t)$$

Static & Dynamic Systems Contd.

- A dynamic system possesses memory
- It has the storage devices
- A system is said to possess *memory* if its output signal depends on past values and future values of the input signal

- Examples of memoryless systems:

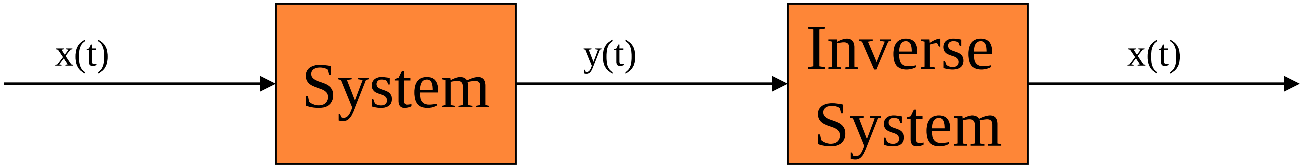
$$y(t) = Rx(t) \qquad \text{or} \qquad y[n] = (2x[n] - x^2[n])^2.$$

- Examples of systems with memory:

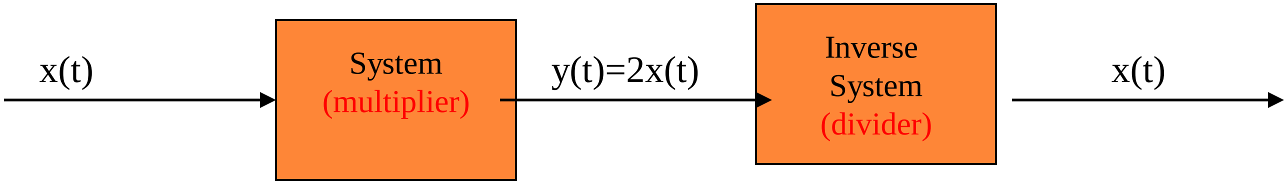
$$y(t) = \frac{1}{C}\int_{-\infty}^{t} x(\tau)d\tau \qquad \text{or} \qquad y[n] = x[n-1].$$

Invertible & Inverse Systems

- If a system is invertible it has an Inverse System



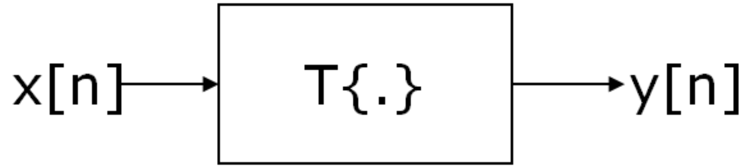
- Example: y(t)=2x(t)
  - System is invertible→ must have inverse, that is:
  - For any x(t) we get a distinct output y(t)
  - Thus, the system must have an Inverse
    - x(t)=1/2 y(t)=z(t)



DISCRETE-TIME SYSTEMS

- A Discrete-Time System is a mathematical operation that maps a given input sequence x[n] into an output sequence y[n]

$$y[n] = T\{x[n]\}$$



Example:

Moving (Running) Average

$$y[n] = x[n] + x[n-1] + x[n-2] + x[n-3]$$

Maximum $\quad y[n] = \max\{x[n], x[n-1], x[n-2]\}$

Ideal Delay System $\quad y[n] = x[n-n_o]$

Memoryless System

A system is memoryless if the output y[n] at every value of n depends only on the input x[n] at the same value of n

Example :

Square $$y[n] = (x[n])^2$$

Sign $$y[n] = \mathrm{sign}\{x[n]\}$$

counter example:

Ideal Delay System $$y[n] = x[n - n_o]$$

Linear Systems

- Linear System: A system is linear if and only if

$$T\{x_1[n] + x_2[n]\} = T\{x_1[n]\} + T\{x_2[n]\} \quad \text{(additivity)}$$

and

$$T\{ax[n]\} = aT\{x[n]\} \quad \text{(scaling)}$$

Example: Ideal Delay System

$$y[n] = x[n - n_o]$$

$$\begin{aligned}
T\{x_1[n] + x_2[n]\} &= x_1[n - n_o] + x_2[n - n_o] \\
T\{x_2[n]\} + T\{x_1[n]\} &= x_1[n - n_o] + x_2[n - n_o] \\
T\{ax[n]\} &= ax_1[n - n_o] \\
aT\{x[n]\} &= ax_1[n - n_o]
\end{aligned}$$

Time-Invariant Systems

Time-Invariant (shift-invariant) Systems

$$y[n] = T\{x[n]\} \Rightarrow y[n-n_o] = T\{x[n-n_o]\}$$

A time shift at the input causes corresponding time-shift at output

Example: Square

$$y[n] = (x[n])^2$$

Delay the input the output is $y_1[n] = (x[n-n_o])^2$

Delay the output gives $y[n - n_o] = (x[n-n_o])^2$

Counter Example: Compressor System

$$y[n] = x[Mn]$$

Delay the input the output is $y_1[n] = x[Mn-n_o]$

Delay the output gives $y[n - n_o] = x[M(n-n_o)]$

Causal System

A system is causal iff it's output is a function of only the current and previous samples

Examples: Backward Difference

$$y[n] = x[n] - x[n-1]$$

Counter Example: Forward Difference

$$y[n] = x[n+1] + x[n]$$

STABLE SYSTEM

Stability (in the sense of bounded-input bounded-output BIBO). A system is stable iff every bounded input produces a bounded output

$$|x[n]| \le B_x < \infty \Rightarrow |y[n]| \le B_y < \infty$$

Example: Square $y[n] = (x[n])^2$

if input is bounded by $|x[n]| \le B_x < \infty$

output is bounded by $|y[n]| \le B_x^2 < \infty$

Counter Example: Log $y[n] = \log_{10}(|x[n]|)$

even if input is bounded by $|x[n]| \le B_x < \infty$

output not bounded for $x[n] = 0 \Rightarrow y[0] = \log_{10}(|x[n]|) = -\infty$

THANKS