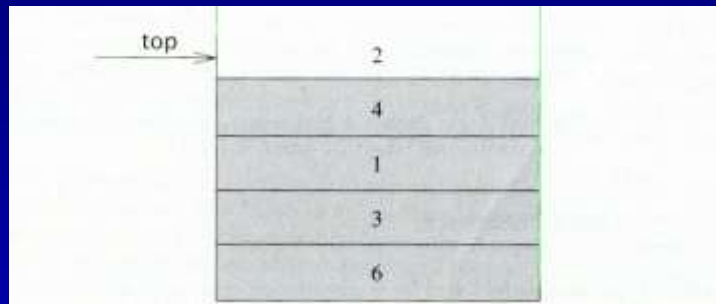


Stacks

Data Structures

What is a stack?

- A stack is a list with the restriction
 - that insertions and deletions can only be performed at the *top* of the list



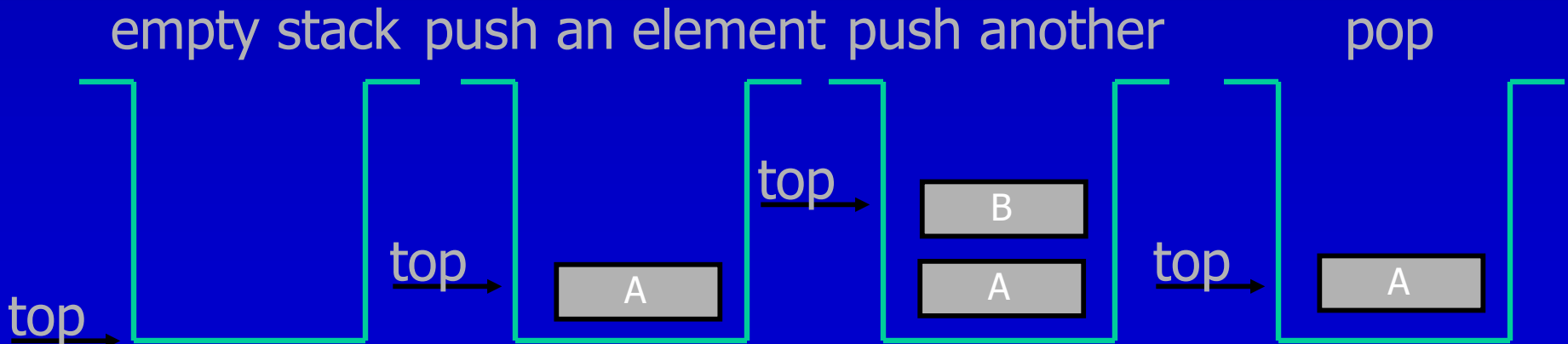
- The other end is called bottom
- Fundamental operations:
 - Push: Equivalent to an insert
 - Pop: Deletes the most recently inserted element
 - Top: Examines the most recently inserted element

Stack

- Stacks are less flexible
 - ✓ but are more efficient and easy to implement
- Stacks are known as LIFO (Last In, First Out) lists.
 - The last element inserted will be the first to be retrieved

Push and Pop

- Primary operations: Push and Pop
- Push
 - Add an element to the top of the stack
- Pop
 - Remove the element at the top of the stack



Implementation of Stacks

- Any list implementation could be used to implement a stack
 - Arrays (static: the size of stack is given initially)
 - Linked lists (dynamic: never become full)
- We will explore implementations based on array and linked list
- Let's see how to use an array to implement a stack first

Stack class

- Attributes of Stack
 - `maxTop`: the max size of stack
 - `top`: the index of the top element of stack
 - `values`: point to an array which stores elements of stack
- Operations of Stack
 - `IsEmpty`: return true if stack is empty, return false otherwise
 - `IsFull`: return true if stack is full, return false otherwise
 - `Top`: return the element at the top of stack
 - `Push`: add an element to the top of stack
 - `Pop`: delete the element at the top of stack
 - `DisplayStack`: print all the data in the stack

Push Stack

- `void Push(const double x);`
 - Push an element onto the stack
 - If the stack is full, print the error information.
 - Note `top` always represents the index of the top element. After pushing an element, increment `top`.

```
void Stack::Push(const double x) {  
    if (IsFull())  
        cout << "Error: the stack is full." << endl;  
    else  
        values[++top] = x;  
}
```

Pop Stack

- `double Pop()`
 - Pop and return the element at the top of the stack
 - If the stack is empty, print the error information. (In this case, the return value is useless.)
 - Don't forget to decrement `top`

```
double Stack::Pop() {  
    if (IsEmpty()) {  
        cout << "Error: the stack is empty." << endl;  
        return -1;  
    }  
    else {  
        return values[top--];  
    }  
}
```


Stack Top

- `double Top()`
 - Return the top element of the stack
 - Unlike `Pop`, this function does not remove the top element

```
double Stack::Top() {  
    if (IsEmpty()) {  
        cout << "Error: the stack is empty." << endl;  
        return -1;  
    }  
    else  
        return values[top];  
}
```

Balancing Symbols

- To check that every right brace, bracket, and parentheses must correspond to its left counterpart
 - e.g. `[()]` is legal, but `[(])` is illegal
- Algorithm
 - (1) Make an empty stack.
 - (2) Read characters until end of file
 - i. If the character is an opening symbol, push it onto the stack
 - ii. If it is a closing symbol, then if the stack is empty, report an error
 - iii. Otherwise, pop the stack. If the symbol popped is not the corresponding opening symbol, then report an error
 - (3) At end of file, if the stack is not empty, report an error

Postfix Expressions

- Calculate $4.99 * 1.06 + 5.99 + 6.99 * 1.06$
 - Need to know the precedence rules
- Postfix (reverse Polish) expression
 - $4.99\ 1.06\ *\ 5.99\ +\ 6.99\ 1.06\ *\ +$
- Use stack to evaluate postfix expressions
 - When a number is seen, it is pushed onto the stack
 - When an operator is seen, the operator is applied to the 2 numbers that are popped from the stack. The result is pushed onto the stack
- Example
 - evaluate $6\ 5\ 2\ 3\ +\ 8\ *\ +\ 3\ +\ *$
- The time to evaluate a postfix expression is $O(N)$
 - processing each element in the input consists of stack operations and thus takes constant time